APPENDIX G
ACTIVITY #2 – THE PATH OF THE SUN IN THE SKY

Though several options are available to portray the path of the Sun in the Sky, most will find it easier overall to:

- First, calculate the altitude and azimuth of the Sun at each point of the analemma (Steps 1 through 4, below).
- Second, using the equations for conversion of alt-azimuth coordinates to equatorial coordinates, convert the alt-azimuth coordinates to Declination and Hour Angle (Step 5, below).

**Step 1:** Continue with the coordinate system introduced in Activity #1:
- P(0,0,0) at the opening of the enclosure / tip of the gnomon.
- The x-axis as east / west (positive being eastward).
- The y-axis as north / south (positive being northward).
- The z-axis as up / down (positive being upward).

Note that the analemma is in the x-/y-plane at \( z = -h \).

**Step 2:** Digitize the analemma. This will generally be done by scanning or photographing the analemma. Be sure to allow for the following:
- The location of the point directly below the opening of the enclosure / tip of the gnomon … this point will be referred to as P(0,0,-h). Accurate measurement of distances relative to this point is crucial to the calculations that follow.
• If photographing the analemma, take the image from directly above (i.e., perpendicular to) the analemma using as long a focal length as possible to minimize distortion. Wide-angle lenses should not be used. The “long dimension” of the analemma should be aligned with the width of the camera’s field of view as well as possible.
• Scaling the scan / image must be included, so conversion from locations in the image to measured distances can be made.

**Step 3:** Translate the zero-point on the image from Step 2 (generally the upper left corner) to P(0,0,-h) ... i.e., the point directly below the opening in the enclosure / tip of the gnomon in the coordinate system described in Step 1.

When translating the origin of a coordinate system to a point having the coordinates P(h,k) within that system, then the coordinates of a point P(x,y) will change to:

\[
x' = x - h \\
y' = y - k
\]

where: x and y refer to the original (pre-translation) coordinates. 
x’ and y’ refer to the post-translation coordinates.

Note: This relationship assumes an x-positive to the right / y-positive up orientation. If the image’s coordinate system has different orientation, corrective measures will have to be taken.

**Step 4:** For each point of the analemma, calculate the altitude-azimuth coordinates:
• Calculate new x and y values based on translation of the axes to the point on the floor of the enclosure directly below the opening (i.e., P(0,0,-h)).
• Provide for calculation new x and y values based on the rotation of the axes around the z-axis. (This is a correction for magnetic deviation, improper alignment of the observing apparatus along true north / south, or if photographing the analemma, not properly aligning the analemma within the camera’s field of view.) Initially, this angle of rotation will be set to 0° (i.e., not rotated).

When rotating a coordinate system an angle \(\alpha\) (alpha) around its origin,

\[
X = x \cdot \cos(\alpha) + y \cdot \sin(\alpha) \\
Y = -x \cdot \sin(\alpha) + y \cdot \cos(\alpha)
\]

where: x and y refer to the original (pre-rotation) coordinates. 
X and Y refer to the post-rotation coordinates.

Note: \(\alpha\) is positive in the counter-clockwise direction
• Calculate the angle off the x=0 plane (a.k.a., the y-/z-plane, which contains the Celestial Meridian). Numerically, it is the arctan(x/y); above the opening, it is the angle, \( \phi \) (phi).

   Note that the Sun’s Azimuth is: \( 180^\circ + \phi \)

• Calculate the angle off the x-/y- ("horizontal") plane. Numerically, it is:
   \[ \text{Arctan}\left(\frac{h}{\sqrt{x^2+y^2}}\right) \]

   Above the opening, it is the angle, \( \theta \) (theta).

   Note that \( \theta \) (theta) is also the Sun’s Altitude.

**Step 5:** Calculate the Declination and Hour Angle for the Sun at each reading in the analemma. The equations are presented in *Practical Astronomy With Your Calculator*, by Peter-Duffett Smith §26.

\[
\sin(\delta) = \sin(a)\sin(\varphi) + \cos(a)\cos(\varphi)\cos(A) \\
\cos(H) = \left( \frac{\sin(a) - \sin(\varphi)\sin(\delta)}{\cos(\varphi)\cos(\delta)} \right)
\]

where, \( a = \) altitude of the Sun (from Step 4, above).
\( A = \) Azimuth of the Sun (from Step 4, above).
\( \delta = \) declination of the Sun.
\( \varphi = \) Latitude of the Observer (from Activity #1).
\( H = \) the Hour-Angle between the Sun and the Meridian.

**Step 6:** Plot the Declination (vertical axis) vs. Hour Angle (horizontal axis)

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