Though several options are available to portray the path of the Sun in the Sky, most will find it easier overall to:

- First, calculate the altitude and azimuth of the Sun at each point of the analemma (Steps 1 through 4, below).
- Second, using the equations for conversion of alt-azimuth coordinates to equatorial coordinates, convert the alt-azimuth coordinates to Declination and Hour Angle (Step 5, below).

Step 1: Continue with the coordinate system introduced in Activity #1:
- \( P(0,0,0) \) at the opening of the enclosure / tip of the gnomon.
- The x-axis as east / west (positive being eastward).
- The y-axis as north / south (positive being northward).
- The z-axis as up / down (positive being upward).

Note that the analemma is in the x / y plane at \( z = -h \).

Step 2: Digitize the analemma. This will generally be done by scanning or photographing the analemma. Be sure to allow for the following:
- The location of the point directly below the opening of the enclosure / tip of the gnomon … this point will be referred to as \( P(0,0,-h) \). Accurate measurement of distances relative to this point is crucial to the calculations which follow.
- If photographing the analemma, take the image from directly above (i.e., perpendicular to) the analemma using as long a focal length as possible.
to minimize distortion. Wide-angle lenses should not be used. The “long dimension” of the analemma should be aligned with the width of the camera’s field of view as well as possible.

- Scaling the scan / image must be included, so conversion from locations in the image to measured distances can be made.

**Step 3:** Translate the zero-point on the image from Step 2 (generally the upper left corner) to P(0,0,-h) … i.e., the point directly below the opening in the enclosure / tip of the gnomon in the coordinate system described in Step 1.

When translating the origin of a coordinate system to a point having the coordinates P(h,k) within that system, then the coordinates of a point P(x,y) will change to:

\[
x' = x - h \\
y' = y - k
\]

where: x and y refer to the original (pre-translation) coordinates. \(x'\) and \(y'\) refer to the post-translation coordinates.

Note: This relationship assumes an x-positive to the right / y-positive up orientation. If the image’s coordinate system has different orientation, corrective measures will have to be taken.

**Step 4:** For each point of the analemma, calculate the altitude-azimuth coordinates:

- Calculate new x and y values based on translation of the axes to the point on the floor of the enclosure directly below the opening ( P(0,0,-h) ).
- Provide for calculation new x and y values based on the rotation of the axes around the z-axis. (This is a correction for magnetic deviation, improper alignment of the observing apparatus along true north / south, or if photographing the analemma, not properly aligning the analemma within the camera’s field of view.) Initially, this angle of rotation will be set to 0° (i.e., not rotated).

When rotating a coordinate system an angle \(\alpha\) (alpha) around its origin,

\[
X = x \cdot \cos(\alpha) + y \cdot \sin(\alpha) \\
Y = -x \cdot \sin(\alpha) + y \cdot \cos(\alpha)
\]

where: x and y refer to the original (pre-rotation) coordinates. \(X\) and \(Y\) refer to the post-rotation coordinates.

Note: \(\alpha\) is positive in the counter-clockwise direction.
Calculate the angle off the x=0 plane (a.k.a., the y,z plane, which contains the Celestial Meridian). Numerically, it is the $\arctan(x / y)$; above the opening, it is the angle, $\phi$ (phi). Note that the Sun’s Azimuth is $180^\circ + \phi$.

Calculate the angle off the x, y (“horizontal”) plane. Numerically, it is $\arctan( h/\sqrt{x^2+y^2} )$; above the opening, it is the angle, $\theta$ (theta). Note that this is also the Sun’s Altitude.

**Step 5:** Calculate the Declination and Hour Angle for the Sun at each reading in the analemma. The equations are presented in *Practical Astronomy With Your Calculator*, by Peter-Duffett Smith §26.

\[
\sin(\delta) = \sin(a)\sin(\phi) + \cos(a)\cos(\phi)\cos(A)
\]

\[
\cos(H) = \frac{(\sin(a) - \sin(\phi)\sin(\delta))}{\cos(\phi)\cos(\delta)}
\]

where, 
- $a$ = altitude of the Sun (from Step 4, above).
- $A$ = Azimuth of the Sun (from Step 4, above).
- $\delta$ = declination of the Sun.
- $\phi$ = Latitude of the Observer (from Activity #1).
- $H$ = the Hour-Angle between the Sun and the Meridian.

In the above equation for $\cos(H)$, the values of $H$ will always be positive. To determine the sign of $H$ use the following equation:

\[
\sin(H) = -\sin(A)\cos(a) / \cos(\delta)
\]

For a good description of how the above equations are derived see the following link: [http://star-www.st-and.ac.uk/~fv/webnotes/chapter7.htm](http://star-www.st-and.ac.uk/~fv/webnotes/chapter7.htm)

**Step 6:** Plot the Declination (vertical axis) vs. Hour Angle (horizontal axis).

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